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Investigation of the role of temperature fluctuations on spectral line shapes

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Abstract

In low frequency turbulent plasmas, the profile of a Doppler broadened line shape is linked to an effective velocity distribution function (VDF) of the emitters. This distribution is obtained through both a spatial and time average of the local velocity distribution. We study the case of the Balmer α line of deuterium (Dα), in typical conditions encountered in ionising edge plasmas ($N_e = 10^{18}$–$10^{19}\,\mathrm{m}^{-3}$ and T_e–T_i = 10–100 eV). We focus this analysis on ion temperature fluctuations, and assume that the acquisition time of the spectrometer is much larger than the typical turbulent time scale. The resulting expression of the effective VDF involves the temperature probability distribution function (PDF). Temperature fluctuations are found to strongly affect the tails of the effective VDF (i.e. line wings), especially when their PDF has a heavy tail. This opens the way for a new spectroscopic analysis of edge plasma turbulence.

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1. Introduction

Edge plasmas of fusion devices are known to be strongly turbulent, with fluctuation rates rising up to several tenths of percents [1]. Characterizing the statistical properties of these turbulent fluctuations, such as their Probability Distribution Function (PDF), is of primary importance. Indeed, strong deviations of these PDFs from gaussianity are associated to intermittent or bursty transport events which might govern the transport of energy and particles to the wall [2]. Therefore, many experimental works have been devoted to time resolved studies of electron density and temperature fluctuations, using either Langmuir probes [3,4] or Beam Emission Spectroscopy [5,6].

Spectral line shapes obtained by passive spectroscopy are widely used to diagnose edge plasmas of fusion devices. Studies of the Balmer α of Deuterium (Dα) have for example provided valuable information on sources of neutrals in edge plasmas [7–9]. The potential use of Doppler line shapes to diagnose turbulence has not yet been fully investigated. The need for obtaining acceptable signal to noise ratios precludes time resolution, since acquisition times much larger than the typical turbulent time scale have to be used. In this paper we show that line shapes contain information on the PDF of the fluctuating fields, and that ion temperature fluctuations have to be taken into account in the modelling. Indeed, these can for instance lead to a power-law behavior in

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the line wings when the temperature PDF has an algebraic decay.

2. Measured Profile and effective VDF

In the conditions found in edge plasmas operating in the ionising regime ($N_e = 10^{18}$–$10^{19}\,\mathrm{m}^{-3}$ and T_e–$T_i = 10$–$100\,\mathrm{eV}$), the broadening of low lying lines such as Dα, which will be studied in this paper, is dominated by Zeeman and Doppler effects, at least in the bulk of the line. Indeed, Stark effect governs far line wings, i.e. becomes dominant for wavelength detunings larger than a value $\Delta\lambda_s$, which is essentially a function of the density [10]. Nevertheless, there exists regions of the spectrum for which the orderings $\Delta\lambda_{1/2} < \Delta\lambda < \Delta\lambda_s$ are satisfied, where $\Delta\lambda_{1/2}$ is the Doppler HWHM [11]. These parts of the spectrum, which will be called Doppler line wings in the remainder of the paper, will prove to be of particular interest. In the following, we shall restrict ourselves to detunings such that $\Delta\lambda < \Delta\lambda_s$, and consider the Doppler profile of a single Zeeman component. Let τ_m denote the acquisition time of the spectrometer, and the z axis be chosen along the direction of the line of sight (LOS). If L stands for the length of the emitting zone, the physical intensity measured for a wavelength detuning $\Delta\lambda$ is given by

$$I_m(\Delta\lambda) = \int_0^{\tau_m} \frac{dt}{\tau_m} \int \frac{dz}{L} I_{loc}(\Delta\lambda, z, t) \tag{1}$$

and is obtained from the raw spectrum after deconvolution of the apparatus function. Here I_{loc} is the local profile emitted at time t at location z, which can be written as $I_{loc}(\Delta\lambda, z, t) = B(z,t)I(\Delta\lambda, z, t)$, where B and I are respectively the local normalized brightness and normalized Doppler profile. The local Doppler profile I is related to the emitters VDF integrated over the two directions perpendicular to the LOS through $I(\Delta\lambda, z, t)d\Delta\lambda = f(v_z, z, t)dv_z$. Using this definition together with Eq. (1), we proceed by introducing the emitters effective VDF

$$f_{eff}(v) = \int_0^{\tau_m} \frac{dt}{\tau_m} \int \frac{dz}{L} B(z,t) f(v,z,t). \tag{2}$$

Note that an homogeneous and stationary plasma in which the VDF of the emitters is given by $f_{eff}(v)$ would lead to the same Doppler profile as the actual turbulent plasma. In edge plasmas, there is a separation of scales between turbulence and atomic processes which allows to express f and B in terms of the set of fluid fields $\mathbf{X}(z,t)$ describing turbulence. The brightness B has been calculated using the SOPHIA collisionnal-radiative code [12], and is found to behave quadratically with the electron density $N_e(z,t)$. The electron temperature $T_e(z,t)$ dependence is weak for $T_e(z,t) > 15\,\mathrm{eV}$, and has been neglected. The neutrals VDF f is generally not directly related to the ion VDF f_i. Indeed, there are several sources of neutrals in edge plasmas, and the associated populations are not thermalized with the background ions [7–9]. However, the VDF of the population corresponding to the neutrals created by charge exchange (CX) reactions remain close to f_i, since $\nu_{CX} \gg \tau_{turb}^{-1}$, where ν_{CX} is the CX frequency [13], and τ_{turb} the typical turbulent time scale. For the range of kinetic energies that is of interest here, ν_{CX} can be treated as a constant [14]. In addition, the ion VDF is well approximated by a local Maxwellian since the collision frequency ν_{coll} is such that $\nu_{coll} > \tau_{turb}^{-1}$ and the ion mean free path is shorter than the temperature gradients length. This results in the following expression for the VDF f of the neutrals locally created by charge exchange

$$f(v,z,t) = \sqrt{\frac{m}{2\pi T(z,t)}} \exp\left(-\frac{mv^2}{2T(z,t)}\right), \tag{3}$$

where m is the emitter's mass, and $T(z,t)$ the ion temperature field expressed in units of energy. Note that in this maxwellian approximation the kinetic effects related to density and temperature gradients are neglected. These effects can safely be ignored if the LOS is parallel to the magnetic field lines, since parallel gradients are weak. In addition, for an arbitrary orientation of the LOS it has been shown [15,16] that the neutrals VDF does not deviate significantly from a gaussian in its tails. As a result, these corrections would not strongly affect our conclusions, especially since we will mainly be interested in temperature fluctuations leading to an algebraic decay in the effective VDF tails. In Eq. (3), the component of the fluid velocity along the line of sight (i.e. the parallel velocity) has also been neglected. Even though a complete calculation should take into account velocity and density fluctuations as well as temperature fluctuations, valuable information are gained by considering only the latter. Indeed, if the couplings between these different fields are neglected as a first approximation, density fluctuations do not affect the effective VDF shape since the local VDF given by Eq. (3) does not depend on the density. In addition, an average parallel velocity (i.e. a parallel particle flux) would only result in a shift in the position of the line, whereas parallel velocity fluctuations, which might for instance originate from a Kelvin–Helmoltz instability [17], could be treated using standard results in plasma spectroscopy [18]. On the basis of these assumptions, normalization leads to B = 1, and the effective VDF is given by

$$f_{eff}(v) = \int_0^{\tau_m} \frac{dt}{\tau_m} \int \frac{dz}{L} f(v, T(z,t)). \tag{4}$$

3. Statistical formalism

In order to carry out the calculation of the effective VDF, the solution $T(z,t)$ of the fluid equations is needed. Due to the non-linear nature of these equations, this direct approach would require a numerical treatment. However, such a heavy calculation can be bypassed by taking advantage of the ordering $\tau_m \gg \tau_{turb}$. Indeed, let us consider the sample space variable θ associated to the temperature field $T(z,t)$, and introduce the normalization relation $\int d\theta\delta(\theta - T(z,t)) = 1$ in the expression of the effective VDF. Using the sifting property of the delta function, we obtain for the effective VDF

$$f_{\mathrm{eff}}(v) = \int d\theta \int \frac{dz}{L} \int_0^{\tau_m} \delta(\theta - T(z,t)) \frac{dt}{\tau_m} f(v,\theta). \tag{5}$$

Making use of the ergodic assumption, we introduce the spatially integrated temperature PDF $W(\theta)$

$$W(\theta) = \frac{1}{L} \int dz \langle \delta(\theta - T(z,t)) \rangle, \tag{6}$$

where the brackets denote an ensemble average. The effective VDF is finally given by

$$f_{\mathrm{eff}}(v) = \int_0^{+\infty} d\theta W(\theta) f(v,\theta). \tag{7}$$

The main advantage of this result is that it allows to investigate the shape of the effective VDF resulting from particular choices of the temperature PDF. However, note that this PDF should in principle be determined from the fluid equation governing the temperature field evolution, for instance using the model developed by Pope [19]. The next section will present such an example, but let us first point out a direct connection between our model and the so-called *superstatistics* [20]. Indeed, Eq. (7) is identical to the fundamental equation of this formalism, which establishes that unconventional statistics result from averaging the Boltzman distribution over temperature fluctuations. For instance, if the fluid equations are such that $W(1/\theta)$ is a gamma distribution, then the effective VDF is a Tsallis distribution [21]. In this case, a line shape analysis is not sufficient to distinguish between the actual turbulent plasma and an homogeneous and stationary plasma described by Tsallis non extensive statistical mechanics [21].

4. Application: the Sinai-Yakhot model

Valuable information on the effect of temperature fluctuations on line shapes are gained using for $W(\theta)$ the PDF calculated by Sinai and Yakhot [22]. These authors have considered the case of passive advection in a stochastic incompressible velocity field $\vec{u}$, described by the following equation

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \vec{\nabla} T = \kappa \nabla^2 T. \tag{8}$$

Their method for calculating the PDF is similar to that of Pope. The main difficulty encountered in these approaches is the thorny closure problem which results from the existence of correlations between T and its gradients. Indeed, the equation governing the temperature PDF time evolution itself involves the PDF of ∇T conditioned to the value of T denoted by P, through the quantity

$$D(\theta) = \frac{1}{\langle T^2 \rangle} \int d\nabla\theta P(\nabla\theta|\theta)(\nabla\theta)^2. \tag{9}$$

In the Sinai Yakhot model, the closure is achieved by using the Taylor development $D(\theta) = 1 + k\theta^2$. Note that the correlations vanish in the limit $k \to 0$, and that k therefore appears as a measure of their strength. The temperature PDF obtained by this approach is given by

$$W(\theta) = \frac{C}{\left(1 + \frac{k}{\sigma}(\theta - \theta_0)^2\right)^{1+\frac{1}{2k}}}, \tag{10}$$

where σ characterizes the width of the distribution, θ_0 is an average temperature and C a normalization constant. This distribution is plotted on Fig. 1 for $\theta_0 = 30$ eV, $k = 10$ and $\sigma = 20,100,500$. The resulting effective VDFs are plotted in logarithmic scale on Fig. 2 together with the Maxwellian at 30 eV, which corresponds to the case of a quiescent plasma. The effective VDF stays close to this Maxwellian, except in the tails where significant deviations are observed. Indeed, it is easily verified that a power law behavior of exponent α for the temperature PDF leads to an algebraic decay of exponent $2\alpha - 1$ for the effective VDF tails (i.e. for the wings of the line profile). An elementary study of the integral defining the effective VDF shows that the results obtained above are general: temperature fluctuations are negligible if

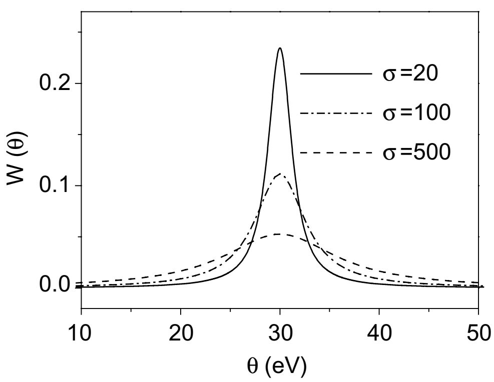

Fig. 1. Plot of the Sinai-Yakhot temperature PDF for $\theta_0 = 30$ eV, $k = 10$ and different values of σ: $\sigma = 20$ (solid line), $\sigma = 100$ (dash-dotted line), $\sigma = 500$ (dotted line).

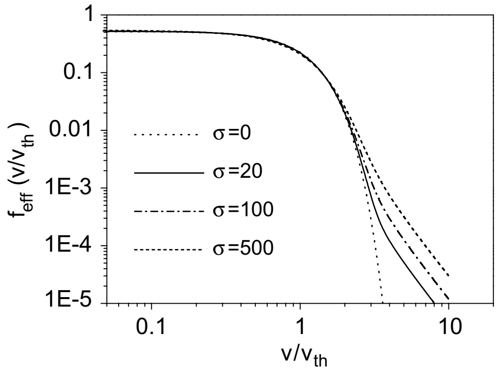

Fig. 2. Plot of the effective VDF in the case of temperature fluctuations described by the Sinai-Yakhot model for $\theta_0 = 30\,\mathrm{eV}$, $k = 10$, and $\sigma = 20$ (solid line), 100 (dash-dotted line), 500 (short-dashed line). The Maxwellian at 30 eV is also plotted (dotted line), and corresponds to the limiting case in which there are no fluctuations ($\sigma = 0$). The bulk of the effective VDF is very close to the Maxwellian, while its tails exhibit a power law behavior. The wider the temperature PDF, the larger the deviations.

the study is restricted to the bulk of the line, and therefore do not impair routine diagnostics based on line shapes. Conversely, line wings are found to deviate from the Maxwellian which would be observed in a quiescent plasma, all the more as the temperature PDF tail decreases slowly. This opens the way for a possible new diagnostic of the statistical properties of ion temperature fluctuations based on line wings studies. Such a technique would require spectra with large signal to noise ratio, which could in principle be obtained by averaging over successive spectra in a stationary discharge. A comparison between experiment and theory should rely on a refined model, retaining not only temperature fluctuations, but also velocity and density fluctuations, as well as the couplings between these different fields. At the present time, a numerical approach (either solving the fluid equations to obtain time histories of the different fields, or working at the PDF level) seems best suited to address these issues.

5. Conclusion

In this paper we have considered the role of low frequency turbulence, ubiquitous in edge plasmas of fusion devices, on Doppler broadened spectral line shapes. The notion of emitter effective velocity distribution function, which is best suited to discuss the problem and in particular the links to unconventional statistics, has been introduced. We have studied a population of neutrals created by charge exchange reactions, whose VDF is closely related to that of the ions, and then focused on ion temperature fluctuations. In the case relevant to most experiments, the acquisition time of the spectrometer is much larger than the typical turbulent time scale, and the effective VDF is determined by the temperature fluctuation PDF. This PDF should in principle be calculated from the temperature fluid equation. In the case of the PDF obtained by Sinai and Yakhot, it is found that the bulk of the line is not altered by temperature fluctuations, whereas line wings undergo significant changes. These results, which are easily generalized to other PDFs, establish a link between passive spectroscopy and turbulence in edge plasmas.

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